# FINITE ELEMENT MODELLING OF INFLUENCE OF GLOBAL SHAPE AND INTERNAL STRUCTURE OF APRICOT ON THE RESONANT FREQUENCY.

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## Abstract

Finite element models were developed using LUSAS finite element system to study the vibration responce spectrum of apricot to sonic excitation. Apricot was modelled as an spherical and non-spherical real shape models consist of skin, flesh and stone or homogeneous material. Simulation results show that first six resonant frequencies are associated with mode shapes which perform rigid body motions. These frequencies are not affected by the model structure. Higher resonant frequencies are affected by the model structure as regards the amplitudes. The frequency peak positions were only slightly changed. The square of the second resonant frequency was linearly related to Young's modulus of the fruit. The modelling approach and elastic properties were validated using laser doppler vibrometry measurement.

**Key words:** finite element modelling, apricot, resonant frequency

## Introduction

A number of methods for quality evaluation of agricultural products have been developed over the past several decades. These methods are based on the detection of various physical properties which correlate with certain quality factors of the products. Non-destructive optical methods which utilize high-speed optical detection and computer data processing are suitable methods (Brozman, 2000). A significant amount of research has been conducted on sonic firmness measurement of fruits (Abbott et al. 1968, Yamamoto et al. 1980, Armstrong et al. 1997).

In the research presented here, special attention is given to resonant frequency estimation of apricot using finite element method. A number of researchers have conducted theoretical analysis to better understand the basic principle of dynamic vibrations of the fruit and to determine various factors affecting sonic firmness measurement. Cook and Rand (1973) studied the vibrational behavior of apples that were considered to be elastic spheres composed of three homogeneous components, the skin, flesh and core. They found that there are two classes of vibrational modes in apples, i.e. the spheroidal and torsional. Chen and De Baerdemaeker (1993) applied the finite element method to analyze the vibrational modes in fruits. They reported that there are three classes of vibrational modes, torsional, the first-type spheroidal and the second-type spheroidal. Lu and Abbott (1996) found that the second-type spheroidal modes are non-axisymetric and most of the vibrational modes between 0 and 2000Hz belong to this class.

However, in the case of apricot, a better understanding of the vibrations measured by laser doppler vibrometry is needed because of the internal structure – flesh, stone. Modal analysis of homogeneous elastic spheres that substituted for fruits have been extensively examined in the past by Yong and Bilanski (1979) and Kimmel et al. (1992). Wu et al. (1994) developed a finete element model based on geometry of real apple and conducted harmonic analysis. Theoretical analysis have been mainly confined to modal analyses under free vibration or harmonic responses of fruits without consideration of the actual boundary conditions imposed on the fruit. Experimental results showed that vibration response spectrum and amplitudes in particular are affected by fruit holding method and location of measurement. It is therefore important to investigate the vibration response spectrum of the fruit under specific experimental set-up configuration.

The overall objective of this study is to develope finite element models to study vibration response spectrum of apricots under specific holding method and with various apricot internal structure and geometry.

## Material and Methods

### Modal Analysis

Modal analysis refers to an experimental or analytical procedure applied in vibrational analysis for describting the dynamic behaviour of continuous mechanical structure with distributed parameters or discrete mechanical structure with lumped parameters. The analysis aims to define the basic deformation shapes (mode shapes) of a mechanical structure when excited at one of its natural frequencies.

Since the mathematical formulations for the stated problem are complex and cannot be solved analytically, the finite element method (FEM) was used to obtain solution. The general procedure for the FEM formulation is documented in many references (e.g. Petyt, 1990, FEA, 1999). The equations of dynamic equilibrium of an elastic discretised body can be written in matrix form as

$$M\,\ddot{u} + C\,\dot{u} + K\,u = R_{(t)} \qquad (1)$$

where ű, ú and u are the acceleration, velocity and displacement vectors respectively, R(t) is the force vector, M, C and K are the mass, damping and stiffness matrices respectively.

For natural frequency analysis, we consider the damping C and the external force R(t) both to be zero. Each freedom is assumed to excite harmonic motion in phase with all other freedoms

$$a = a\sin\omega t \quad \text{and} \quad \ddot{a} = -\omega^2\, a\sin\omega t \qquad (2)$$

where ω is the circular frequency. Then (2) yields

$$\left(K - \lambda_i M\right)a = 0 \quad \text{where} \quad \lambda_i = \omega_i^2 \qquad (3)$$

This is equivalent to the generalised eigen-problem defined by

$$K\Phi = \Lambda M\Phi \qquad (4)$$

where Λ is a diagonal matrix containingthe eigenvalues λ(i) and Φ contains the corresponding eigenvectors ϕ(i).

Neglecting the trivial solution Φ=0 the values of λ(i) and ϕ(i) that satisfy equation (4) are defined as the eigenvalues and eigenvectors of the problem respectively, which are evaluated using subspace iteration (FEA, 1999). The eigenvalues and eigenvectors are the resonant vibrational frequencies and the corresponding mode shapes of a structure respectively.

**Physical Experiment**

In the experiment two groups of eight Barbora apricots of approximately equal size were selected from harvested fruit. The measurement of vibration response spectrum was performed by the laser Doppler vibrometry. During measurement the apricot was put on the polystyrene base. A more detailed description of the instrument and method is given by Brozman (Brozman, 2000). In this way the displacement frequency response at the measured point was obtained as a reference for virtual model validation.

**Virtual Experiment**

For the virtual experiment an apricot was randomly selected from groups used in laser vibrometry measurement. The LUSAS Finite Element System was used to obtain a virtual experiment simulating the physical experiment. LUSAS is an associative feature-based modeller. The model geometry is entered in terms of features which are sub-divided into finite elements in order to perform analysis. Increasing the discretisation of the features will usually result in an increase in accuracy of the solution, but with a corresponding increase in solution time and disk space required. The features in LUSAS form a hierarchy, that is volumes are comprised of surfaces, which in turn are made up of lines which are defined by points. A LUSAS model is a graphical representation consisting of geometric features (points, lines, surfaces, volumes) and assigned attributes (materials, loading, supports, mesh).

Two virtual models were developed. An idealized axisymetric (spherical) model and a non-axisymetric real shaped model. A model was created with the following modelling considerations:

- strict basic SI units (N,m,kg) should be used
- a relatively coarse mesh can be used since stress output is not required
- no loading was applied to the structure and vibration is due to the mass and stiffness of the structure alone
- eigenvalue control data was included to specify details of the analysis
- there is no damping in a natural frequency analysis

First, the real size geometric structure of chosen apricot was developed including shape of apricot and stone shape and orientation. Secondly, the material properties (isotropic elastic constants) of skin, flesh and stone were defined. Mass density of flesh and stone was detrmined as $(980 \pm 15)kg/m^3$ and $(500 \pm 50)$ $kg/m^3$ respectively. E and ε of the apricot specified part were varied during several repeated simulations. For skin, flesh and stone the Young's modulus was estimated from 10MPa to 20MPa, from 1MPa to 5MPa and from 20MPa to 200MPa respectively. Next, the virtual apricot was discretized into HX20M (FEA, 1999) elements. The discretized structure of real shaped apricot model in a wire and solid (rendered) view is presented in fig.1.

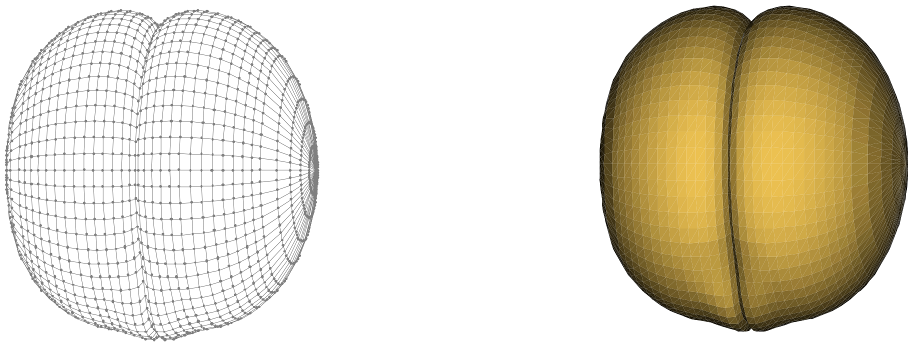


Fig.1 An apricot model meshed by 12300 hexahedral elements in wire and solid view

Final step of the virtual model building was defining the modal analysis by application of the eigenvalue analysis control parameters which represent a load case. The parameters include:

- number of calculated eigenvalues that was set to 10 (in some simulation 20)
- number of starting iteration vector was set to 0
- subspace iteration method to extract the eigenvalues was chosen as more efective than Guyan reduction method for large equation system
- eigenvalue normalisation was set to mass that is essential for subsequent interactive modal dynamics (IMD) analysis (in this case the frequency response function)

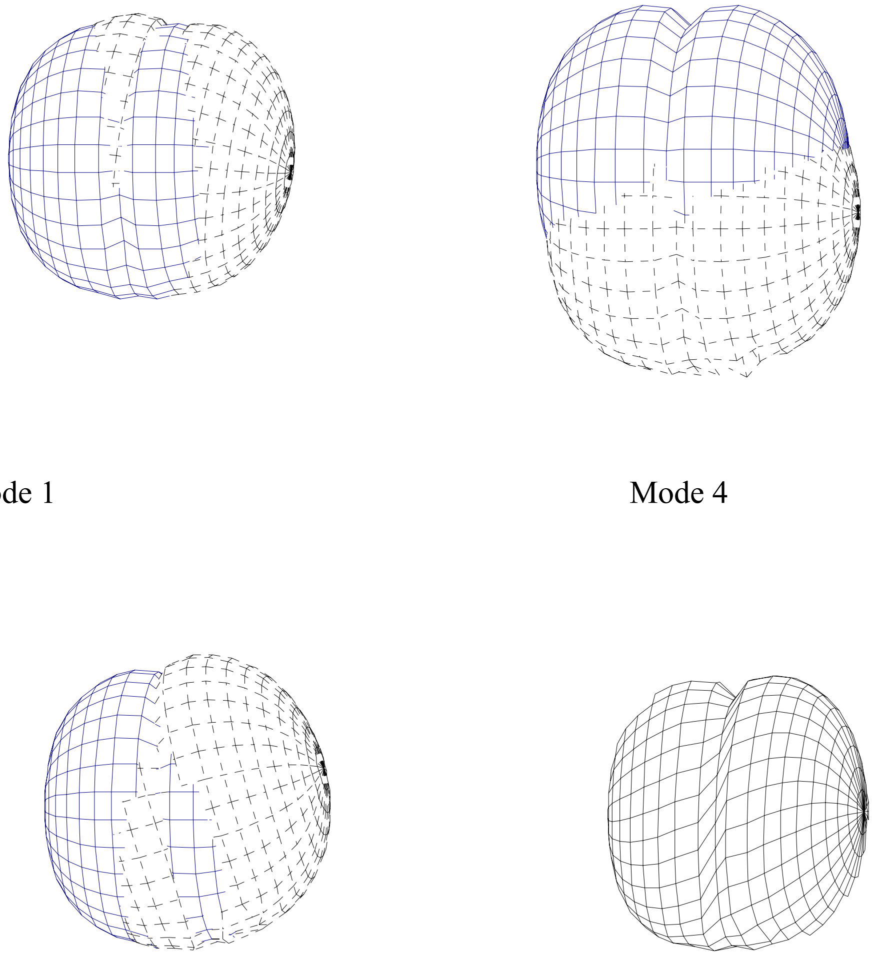
The numerical solution produces a series of eigen pairs. The eigenvalues which indicate frequencies at which the vibration would naturaly occur are output. The eigenvectors give the associated mode shape of vibration. It is important to note that the solved eigenvectors (and hence the resulting mode shape displacements) are normalised and hence may be arbitrarily scaled. It is common for the magnitudes of these quantities to be investigated by running subsequent modal analyses such as forced (harmonic) or spectral response. In this case the resulting eigenvalues were manipulated interactively during results processing using the IMD facility.

**Results and Discussion**

First finite element simulations were performed on an axisimetric model as a test of structure and analysis control. The apricot was considered homogeneous structure of spherical shape of equal material properties chosen for simplicity as E = 1MPa, $\varepsilon = 0.3$, $\rho = 1000 \mathrm{kg/m^3}$. The ten corresponding modes and eigenfrequencies were computed. Then the displacement/frequency response by the IMD was generated for comparison with next simulations and measurement. The data were extracted for the equatorial node of the model that coincides with the measurement point in the experiment.

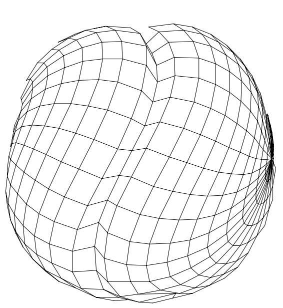
The first six modes can be described as rigid body motion of various types. Modes 1, 2 and 6 can be called wobbling, mode 3 is rigid twist, mode 4 performs bouncing and mode 5 is combination of modes 1 and 3. The higher order mode shapes perform deformation (bending, torsion). The mode shapes are presented in fig.2.

Next, a non-spherical real shape model of apricot was studied during several simulations. The influence of apricot skin and stone on resonant frequency was investigated through three simulations of model. In the first simulation, the apricot was considered homogeneous, the skin, flesh and stone had the same elastic properties (E=1MPa, ε=0.3). In the second simulation, the skin and stone was assumed to have a Young's modulus of 10MPa and 20MPa respectively. In the third simulation was performed by iterative methods in which the Young's modulus was gradually changed to fit the measured resonance spectrum. The Young's modulus of the flesh was determined to 5MPa and the skin and stone remained the same Young's modulus as in second simulation. The shape effect on resonant frequency was examined using the first simulation and simulation of the sphere model. A summary of all simulations for the modal analysis of apricot models is shown in tab.1 and the displacement/frequency response is shown in fig. 3, 4.

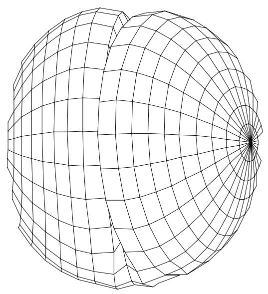


Mode 1 Mode 4

Mode 6 Mode 7



Mode 8



Mode 10

Fig.2 Examples of vibration mode shapes of apricot model. Modes 1, 4, 6 perform rigid body motion and they are presented in comparison with static state (solid line). Next modes are presented in deformed states.

Table 1

| | Spherical model | Firts simulation | Second simulation | Third simulation |
|---|---|---|---|---|
| Mode | Frequency | Frequency | Frequency | Frequency |
| 1 | 6.36 | 6.22 | 6.24 | 13.96 |
| 2 | 15.05 | 10.68 | 10.74 | 24.00 |
| 3 | 18.05 | 21.49 | 21.50 | 48.08 |
| 4 | 54.72 | 58.76 | 59.61 | 132.75 |
| 5 | 54.93 | 93.60 | 93.97 | 209.92 |
| 6 | 58.75 | 103.41 | 103.73 | 231.79 |
| 7 | 353.17 | 335.03 | 338.28 | 753.74 |
| 8 | 364.76 | 366.29 | 371.03 | 822.25 |
| 9 | 365.37 | 374.73 | 380.04 | 842.60 |
| 10 | 365.81 | 376.53 | 381.85 | 848.38 |

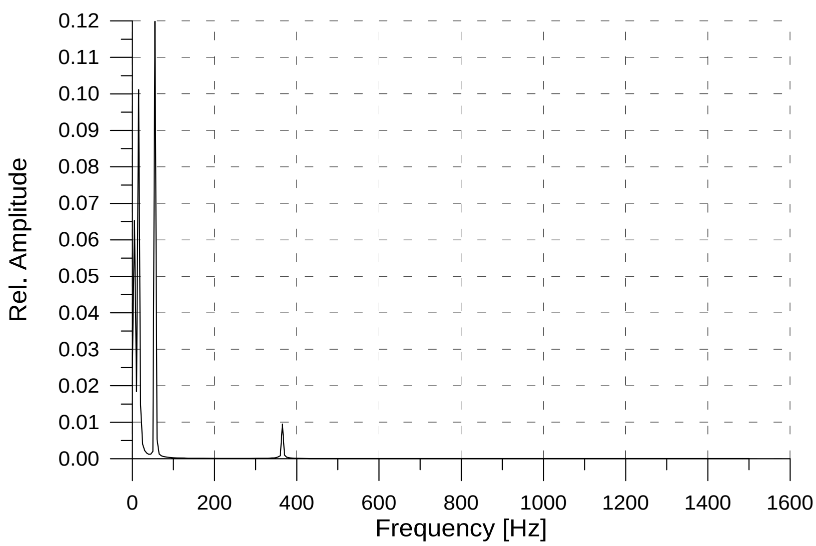



Fig.3 Vibration respose spectrum of spherical apricot model

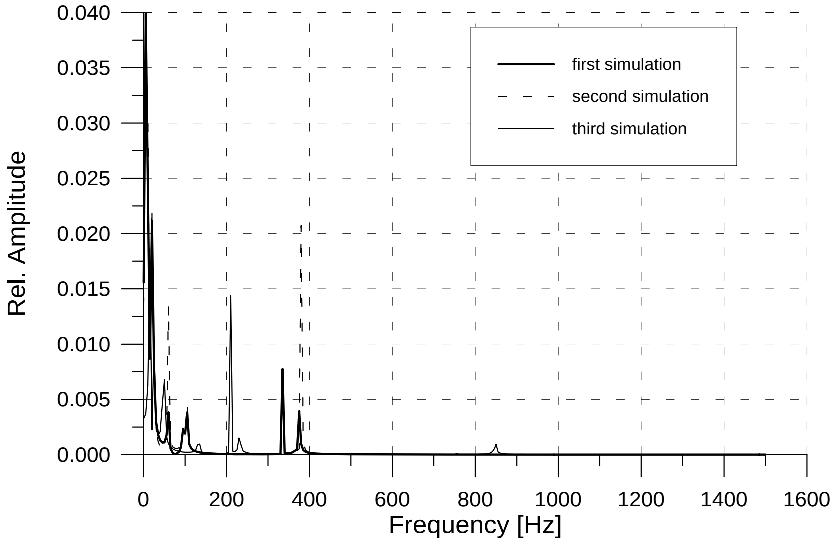



Fig.4 Vibration respose spectrum of real shape apricot models

There was obvious from simulations that as the elastic modulus increased, the resonances shifted to higher frequencies. Overall, the shapes of the responce spectrums were similar up to the sixth resonant frequency, above which the curves showed some different patterns. This indicates that these resonances associated with rigid body mode shapes were independent of the elastic material properties and structure of the apricot model and was likely to be related to the base material (polystyrene was modelled as spring support) and mass of the model.

If the courses of responce spectrums are compared, the presence of skin and stone in the model increases the amplitudes of resonant frequencies of higher modes with deformed mode shapes. Changes of the resonant frequencies are small (not larger that 2%). This indicates that a homogeneous model is a good substitution for real structure model. An ideal shape of model (sphere) had a slightly larger effect to the frequency changes but for most of the frequencies not more than 5%.
Fig.5 shows the relationship between the square of the second resonant frequency (associated with second deformed mode shape) and Young's modulus which was obtained from progress of third simulation. There is a linear relationship. Although next resonant frequencies also change with Young's modulus, the relationship is non-linear. These result demonstrate that the second resonant frequency is a better indicator of firmness.

Responce spectrum obtained from simulation for Young's modulus of 5MPa aproximately coincide the measurement that is presented in fig.6, 7.

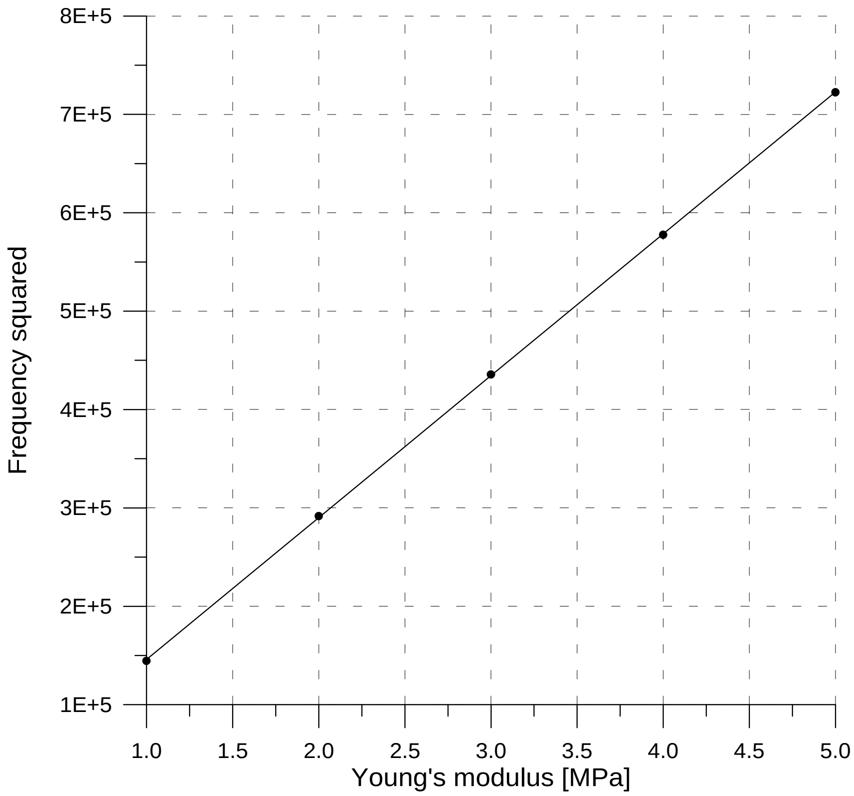



Fig.5 Relationship between the square of the second resonant frequency and Young's modulus.

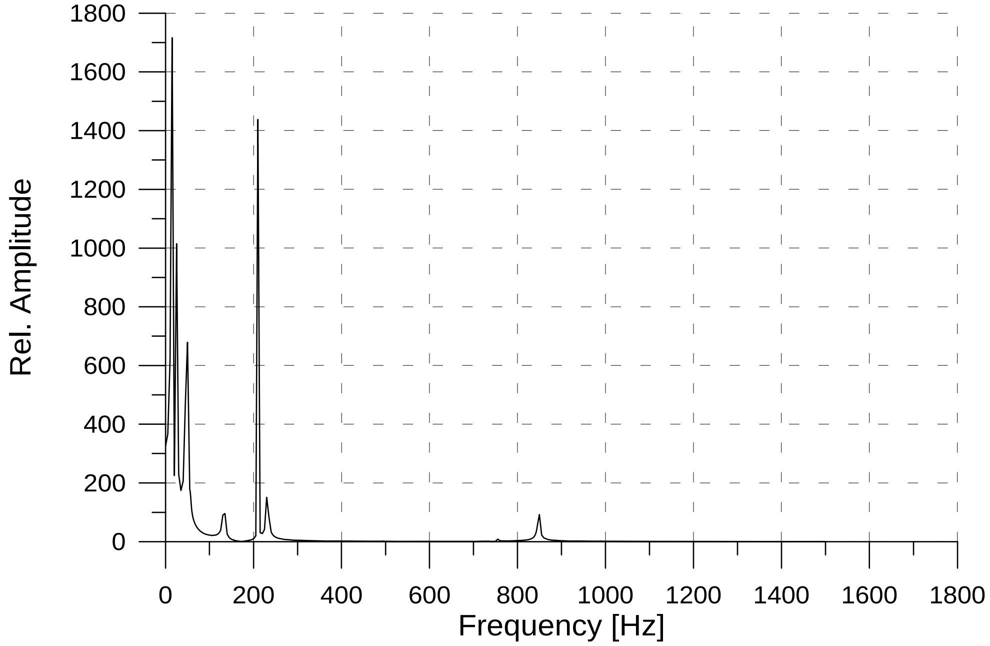



Fig.6 Simulated vibration response spectrum for apricot model with Young 's modulus of 5MPa.

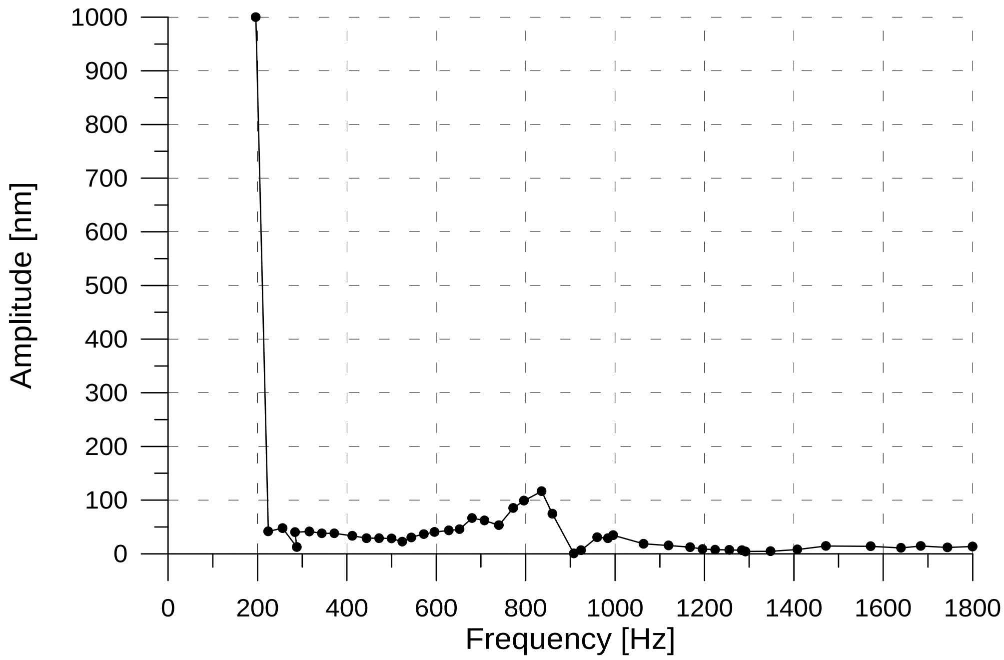



Fig.7 Measured vibration response spectrum of apricot

**Conclusions**

Modal analysis by the finite element method was used to study the vibration measurement on apricot and to optimize a virtual model of apricot. On the basis of the obtained modal analysis information, such as resonant frequencies, Young's modulus estimation, visualisation and animation of the mode shapes, the apricot computer model was developed and the laser doppler vibrometry technique can be optimized for fruit vibration measurement.

The finite element analysis of differently shaped and internally structured apricots revealed that first six vibration modes perform rigid body motions and associated resonant frequencies can not be used for a firmness evaluation. The first distinct resonant frequency peaks of high amplitude was observed too at the laser doppler vibrometry measurement and it is suitable to omit the corresponding frequency range for further measurement. Simulation results showed that resonant frequency associated with second deformed mode shape can be used for estimation of the firmness index or Young's modulus because square of this frequency has linear dependence on the Young's modulus.

The simulation results further demonstrate that global shape and internal structure generally have little influence on the resonant frequencies but the material properties have significant impact on the averall pattern of the frequency spectrum. Thus, tuning the resonant frequency to fit the measured resonant frequency can be used for mechanical property estimation.

**References**

Abbott,J.A., Childers,N.F., Bachman,G.S., Matusik,F.J. (1968). Acoustic vibration for detecting textural quality of apples. Proc.Am.Soc.Hort.Sci. 93, 725-737

Armstrong,P.R., Zapp,H.R., Brown,G.K. (1990). Impulse excitation of acoustic vibrations in apples for firmness determination. Transaction of the ASAE 33(4), 1353-1359.

Brozman,D., Kubik,L. (2000). Measurement of mechanical properties of apples by laser doppler vibrometry. Res. Agr. Eng., 46, 2000 (4), 139-143.

Chen,H., De Baerdemaeker,J. (1993). Modal analysis of the dynamic behavior of pineapples and its relation to fruit fermness. Transaction of the ASAE, 36, 1439-1444.

Cooke,J.R., Rand,R.H. (1973). A mathematical study of resonance in intact fruits and vegetables using a three-media elastic sphere model. J. Agri. Eng. Res., 18(2), 141-157.

Kimmel,E., Peleg,K., Hinga,S. (1992). Vibration modes of spheroidal fruits. J. Agri. Eng. Res., 52, 201-213.

Lu,R., Abbott,J.A. (1996). Finite element analysis of modes of vibration in apples. J.Texture Studies 27(3), 265-286.

Petyt,M. (1990). Introduction to finite element vibration analysis. Cambridge,U.K., Cambridge university press.

Yamamoto,H., Iwamoto,M., Haginuma,S. (1980). Acoustic impulse response method for measuring natural frequency of fruits. J.Texture Studies 11(2), 117-136.

Yong,Y., Bilanski,W. (1979). Modes of vibration of spheroids at the first and second resonance frequencies. Transactions of the ASAE, 1463-1466.

Wu,N., Pitts,M.J., Davis,D.C. (1994). Modelling mechanical behavior of apple with the finite element method. ASAE p. No.94-3585. St.Joseph,Mich.:ASAE

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