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# SEVERAL GEOGEBRA SOLUTIONS FOR MICROECONOMICS

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## Abstract

*Visualization through the use of physical objects is an important method that teachers can use to convey and solidify an understanding of mathematical or economic principles in their students. Creating visual representations for students can open up understanding. The paper deals with the solution of selected microeconomics problems using software GeoGebra. We focused on content of education of the students of Faculty of Economics and Management Slovak University of Agricultural in Nitra. Presented examples can be used for mathematical or microeconomic lessons. Created study material and applets, presented in article, will be available to students in LMS Moodle course "Matematická analýza" (http://moodle.uniag.sk/). We use LMS Moodle to augment face-to-face education. Our research shows that 88% of students believe that online courses are excellent support for the learning process and self-study.*

**Keywords:** *visualization, GeoGebra, mathematics, microeconomics*

**JEL classification:** *A12, A22, I20*

## 1 Introduction

Math is a subject, in which it is aimed to develop analytical thinking skills that could develop students' abstract thinking and allows them understand cause and effect relationship between the events (Yenilmez and Özbey, 2006). In solving word problems in mathematical economics, two different knowledge bases are required: a database of math formulas and a database of economics theories (Shirota, Hashimoto and Stanworth, 2013). Solving a word problem in mathematical economics is nothing more or less than conducting a process of deductive reasoning to find the unknown of the problem (Betsur, 2006). To construct the deductive reasoning process is to collect missing pieces of information from the knowledge bases, to bridge between the given data and the unknown of the problem (Shirota, Hashimoto and Stanworth, 2013).

Visualization of economic concepts through the use of physical objects is an important, and underutilized, method that teachers can use to convey abstract economic concepts with a permanence that lends power to their economic arguments, a method which will help students to understand and remember the concepts more fully (Johnson, 2007). Visualization is no longer related to the illustrative purposes only, but is also being recognized as a key component of reasoning (deeply engaging with the conceptual and not the merely perceptual), problem solving, and even proving (Arcavi, 2003). Why economists build mathematical models? Mathematical representation of all forms is used widely, becoming increasingly important in science as the sophistication level of the models employed rises (Gilbert, 2010). A visual example helps to convey and solidify the fundamental understanding that economists use models to identify and describe key relationships in a world that consists of a complex interrelated web of relationships (Johnson, 2007). Some people may claim to "see" through symbolic forms, regardless of their complexity, but for others, visualization can have a powerful complementary role in the three aspects (Arcavi, 2003), visualization as:

a) a support and illustration of essentially symbolic results,
b) a possible way of resolving conflict between (correct) symbolic solutions and (incorrect) intuitions,
c) a way to help us re-engage with and recover conceptual underpinnings which may be easily bypassed by formal solution.

## 2 Why mathematical software in economical tasks?

Mathematics is an essential element of economics problem solving. The importance of math skills for study success in economics has been widely researched (Arnold and Rowaan, 2014; Arnold and Straten, 2012; Cappellari, Lucifora and Pozzoli, 2012). Students with strong elementary math skills have been performing significantly better in applied contexts which ones improve a student's ability to apply mathematics substantially (Gallo and Johnson, 2008). The application of mathematics and the performing of mathematical modeling and problem solving is suitable means for developing general competencies and attitudes of students. The mathematical software provides visual representation of special problems can also be understood as a cognitive tool important to achieve a higher quality of visual thinking of students and their ability to apply complex mathematical knowledge. In our paper we present economic problems that can be solved from the perspective of the economic or mathematical system and also using software GeoGebra. Understanding given problems from a mathematical point of view it will help to understand the problems in general. We agree with Vallo, Páleniková and Rumanová (2017) that using ICT in education process and dynamic nature of software could help teachers develop necessary students' thinking skills and motivate students to find out new relation between given data.

Models in economics have the wide range of forms including graphs, which are considered an essential tool of economic analysis. Graphs allow understand the links between economic variables and are useful way for their better understanding (Országhová, 2015). Visualization through graphs is an important element of the learning and teaching process, and often provides a critical link between abstract concepts and developing understandings. This is one of the most effective methods to understanding economics models because one picture can spawn a thousand ideas. Visual stimuli can help not only in the learning of economics, but in the retention and application of that knowledge well after our students graduate (Johnson, 2007).

## 3 Some application examples

Solving applied tasks helps create links between study subjects, and furthermore, compile logical associations between theoretical knowledge and practical usage (Hornyák Gregáňová and Országhová, 2017). Application problems develop students' independence, activity and creativity (Pavlovičová and Rumanová, 2012). When students solve application problems, then they learn not only to formulate problems but also solve them in specified contexts and at last formulate the correct conclusions of given problems.

In this section we deal with few possible applications. Defined mathematical-economic issues are solved by software GeoGebra. We focused on content of education of the students of Faculty of Economics and Management Slovak University of Agricultural in Nitra. Presented examples can be used for mathematical or microeconomic lessons. In all problems, we inspired by Agarwala (2008), Barnet, Ziegle and Bylee (2008), Deepashree (2016), Macdonald, Breidenbach and Doetschman (2003), Turnovec (1993).

## 3.1 Budget line and budget constraint

A budget line is a line which shows all possible combinations of two goods that a consumer can buy with his income and price of the commodities. Budget constraint means that total expenditure incurred on the two goods should not be more than the income of the consumer. Graphically, the budget constraint is represented by the budget line and budget set. Budget set is the collection or set of all the possible bundles or combinations of two goods that the consumer can buy with his income and prevailing prices of the commodities. We analyse the changes in budget line in the following problems. The graphical interpretation of these problems is part of the solution. Using GeoGebra in solving those problems could help teachers develop necessary students' thinking skills and open up understanding of students.

***Problem 1:*** The equation for the budget line is

$$p_1x_1 + p_2x_2 = M$$

*where:* $p_1, p_2$ are prices per unit of goods,
$x_1, x_2$ are the quantities of goods,
$M$ is total income of consumer.

Suppose government imposes income tax $u$, a quantity tax $t$ per unit on good $x_1$ and a subsidy of $d$ per unit on $x_2$. Write the budget equation after the introduction of taxes and subsidy and analyse the conditions under which the introduction of taxes and subsidy for households will be disadvantageous in any structure of consumption.

***Solution:*** The budget equation after the introduction of taxes and subsidy is:

$$(p_1 + t)x_1 + \left(p_2 - d\right)x_2 = M - u$$

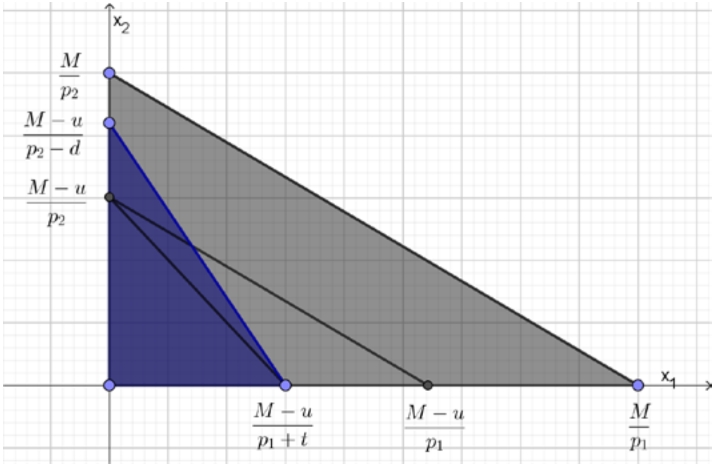
When income decreases due to the income tax, it shifts the budget line leftward. The shifts are parallel. Per unit tax on production of a good makes it dearer and has the same effect as the rise in price of the good, the absolute slope of budget line will be increased:

$$\frac{p_1}{p_2} < \frac{p_1 + t}{p_2}$$

Per unit subsidy on production of a good makes the good cheaper and has the same effect as fall in price of the good, the absolute slope of budget line will be decreased:

$$\frac{p_1}{p_2} > \frac{p_1}{p_2 - d}$$

Figure 1 **Graphical solution of problem 1 through GeoGebra**




If all these changes occur at the same time and must to have only a negative impact on the household, we get:

$$\frac{M-u}{p_2-d} < \frac{M}{p_2} \Rightarrow p_2 * u > M * d$$

(see Figure 1). The consumer spends his entire income on $x_2$ with number of units equal to

$$\frac{M-u}{p_2-d}$$

and on $x_1$ with number of units equal to

$$\frac{M-u}{p_1+t}$$

The budget set will be decreased. From the mathematical expression of the budget set we get:

$$(p_1+t)x_1 - (M-u) > -(p_2-d)x_2$$

$$p_1x_1 + tx_1 - M + u > -p_2x_2 + dx_2$$

$$p_1x_1 + p_2x_2 + tx_1 - (p_1x_1 + p_2x_2) + u > dx_2$$

$$tx_1 + u > dx_2$$

***Problem 2:*** Suppose a consumer has income 600 € and the price of the food he consumes is 4 € per kilogram. Graphically interpret the monthly budget constraint of the consumer. What is the new budget line in case that:

- government allows the consumer to get 20 kg of free food,

- government provides the consumer 150 € per month,
- government allows the consumer food stamp (unlimited), they can to buy food worth 6 € for 3 €.

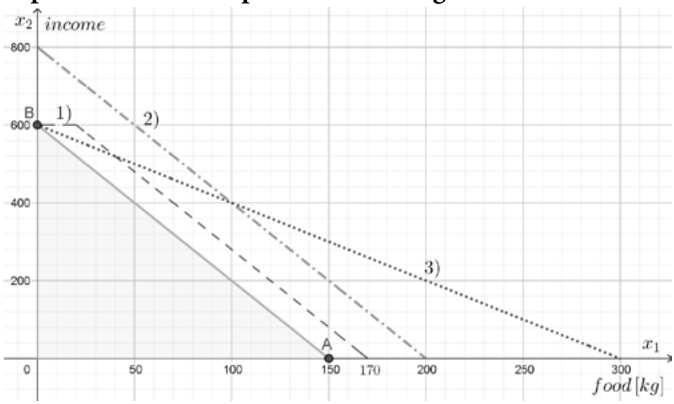
***Solution:*** The budget constraint of this situation can be written as $4x_1 + x_2 \leq 600$ (triangle OAB on Figure 2). If government allows the consumer to get 20 kg of free food, the budget line can be written as

$$x_2 = \begin{cases} 600, x_1 \in \langle 0,20 \rangle \\ 600 - 4(x_1 - 20), x_1 \in \langle 20,170 \rangle \end{cases}$$

(the line 1 on Figure 2).

If government provides the consumer 150 € per month, increase income of consumer *M=600+150=750* and new budget line has the equation $4x_1+x_2=750$ (the line 2 on Figure 2).

Figure 2 **Graphical solution of problem 2 through GeoGebra**




If government allows the consumer food stamp (unlimited), consumer income is not changed, but the amount of food that can be consumed for that income. If consumer spends his whole income on food stamp only then can buy

$$\frac{600}{3} = 200$$

stamps and

$$\frac{200 * 6}{4} = 300kg$$

of food. So the new budget line has the equation

$$2x_1 + x_2 = 600$$

(the line 3 on Figure 2).

## 3.2 The slope of concave curve and opportunity costs

In the real world, we tend to observe increasing opportunity costs which result from specialized resources that are better suited at one productive endeavour than another. The concavity of the production possibilities curve implies increasing opportunity costs of production. For example, we can visualised PPC as a segment of a circle, because in our case is the production potential of both goods the same. The measurement of the opportunity costs is obtained by calculating the slope of the curve at any given point. The slope of concave curve varies from one point to the next. It can be illustrate by drawing a line tangent to the curve at a specific points (see Figure 3). It is clear that the slope of this concave curve is negative and the absolute value of the slope is increasing as the quantity of goods produced increased. This is a reflection of the real world notion of increasing opportunity costs. The opportunity cost refers to the highest valued alternative that is forgone as a result of making a choice.

***Problem 3:*** Company of Woodland can only produce two goods – cheese and wine. Production possibilities curve (PPC) is the segment of the circle mathematical formulated by function

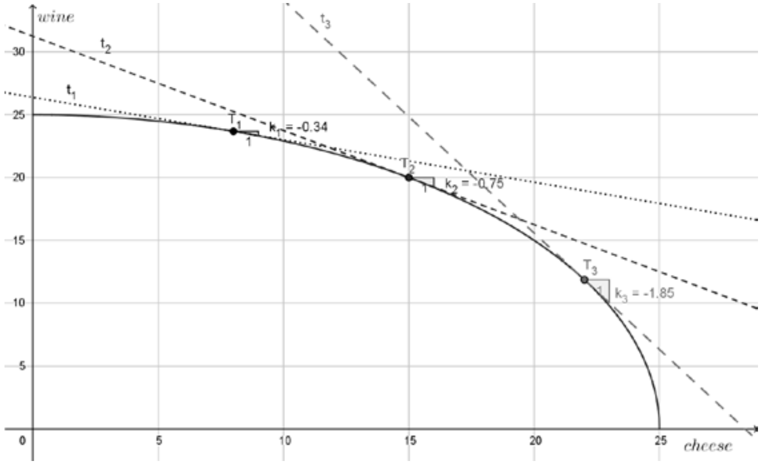
$$f : y = \sqrt{625 - x^2}$$

with domain

$$D(f) = \langle 0{,}25 \rangle$$

Calculate and graphical illustrate the slope of PPC at points $x = 8, 15, 22$. From which of these values of $x$ is the absolute value of the slope greatest? What does this indicate?

Figure 3 **Graphical solution of problem 3 through GeoGebra**




***Solution:*** The slope of the function

$$\frac{\Delta y}{\Delta x}$$

can be approximated by taking the first derivative

$$\frac{dy}{dx}$$

The first derivative

$$y' = \frac{dy}{dx} = \frac{1}{2}(625 - x^2)^{\frac{-1}{2}} * (-2x) = \frac{-x}{\sqrt{625 - x^2}}$$

is negative for 0≤x≤25, so we can say that the slope of the graph is negative. Negative slope tells us that, as *x* increases, *y* decrease. The first derivative of the function *f(x)* is the slope of the tangent line to the function at the point *x*.

When *x*=8, the slope is

$$\frac{dy}{dx} = \frac{-8}{\sqrt{624}} = -0.34$$

When *x*=15, the slope is

$$\frac{dy}{dx} = \frac{-15}{\sqrt{400}} = -0.75$$

When *x*=22, the slope is

$$\frac{dy}{dx} = \frac{-22}{\sqrt{141}} = -1.85$$

The absolute value is greatest when $x$=22. This means that the rate of change of the function is increasing. Each increment of $x$ causes larger and larger increments of $y$.

# 4 Students' views

Only creation of application tasks is not enough. These materials need to be use in the educational process. Teachers of Slovak University of Agriculture in Nitra used blended learning to combines traditional learning and e-learning. We use LMS Moodle (Modular Object Oriented Dynamic Learning Environment) to augment face-to-face education. So, created study material and applets, presented in article, will be available to students and teachers in LMS Moodle course "Matematická analýza" (http://moodle.uniag.sk/).

Students' attitudes to selected method of teaching (blended learning) and using mathematical software or application tasks were determined by questionnaire. We have compiled a questionnaire that students filled out on mathematical exam.

We present results of research, with the participation 603 full-time students (422 women and 181 men): 252 students of Faculty of Biotechnology and Food Sciences (178 women and 74 men), 274 students of Faculty of Economics and Management (189 women and 85 men) and 77 students of Faculty of European Studies and Regional Development (55 women and 22 men). We found that 92% of students have network access at home. We can conclude from the research results that we have chosen the correct method of teaching mathematics, because 57% of students reported that they prefer combined form of teaching mathematics (full-time education supplemented internet course) and 88% of students believe that online courses are excellent support for the learning process and self-study mathematics. 42% of women and 49% of men prefer face to face teaching. But these students also recognize the importance of e-learning, because only 5% of students reported that these courses do not meet the purpose and lack of teacher explanation. We found that students attending courses mainly at the preparation of continuous semester tests and exam (56%). In spite of frequent questions "why we have to study math", only 11% of the students studied the applied problems and only 2% of them all used internet links and read articles dealing with interdisciplinary relationships. There was little interest in material about mathematical software. Only 35% of the students met, and only 12% of them were interested in such materials that they used the mathematical software to solve the seminar work on the course of the function.

# 5 Conclusion

Visual images are a powerful way of transferring mathematical thinking and information because it transmits information in ways that words cannot. The visualized solution is smooth and effective, and, in addition, the process of visualizing provides opportunities for discovery moments. We can say that software GeoGebra is suitable for demonstrations and explorations of the behaviour of the microeconomic characteristics. Through graphical interpretation of the tasks we link aspect of the illustration of the object with aspect of the representation and the description of microeconomic phenomenon and allow development of visual literacy of students. We use LMS Moodle to augment face-to-face education, because our research shows that 88% of students believe that online courses are excellent support for the learning process and self-study. Created study material and applets, presented in article, will be available to students in LMS Moodle course.

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