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PROPOSITIONAL CALCULUS IN TEACHING MATHEMATICAL SUBJECTS AND IN PRAXIS

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Abstract

In our paper we try to point out the most common logical mistakes in mathematical thinking made by students of the Slovak University of Agriculture in Nitra. The mistakes analysis was developed on the basis of students' tests. Students involved in the research were about to take their A-levels in Maths. Tasks in the tests were aimed at the use of an elementary logic – negations, general and existential quantifier in the curriculum of Mathematics at secondary schools and at universities. We tested our main hypothesis, that the involvement of mathematical knowledge into other parts of mathematics will improve a quality of students' knowledge. When formulating the main hypothesis of our research we relied on both, the theoretical knowledge of the issue and the experience based on our own teaching practice. Pedagogical experiment was carried out in two groups – the experimental and the control one.

Keywords: *propositional calculusmathematical thinking, elementary logic, statement*

JEL classification: *C02, C11, I210*

1 Introduction

A statement is an elementary term in both mathematics and logics. Learning to speak correctly means that one should know the basic ways of mathematical logic. Teaching mathematics does not mean only learning mathematical concepts but also developing mathematical thinking. We should point out the interrelationships between different concepts and means of mathematical logic.

Education for skills development must be based not only on efforts of teachers, but also on activities of students. We will focus on teaching mathematical logic and its importance in technical disciplines. Teaching mathematics, in general, contributes to the development of not only mathematical, but also logical thinking. Today, elementary mathematical knowledge and the insights into opportunities it brings are considered to be at least as important as the knowledge of the national history or the laws of physics. Different ways of thinking have come along with the development of mathematics. The issues of math education are still a priority; we talk about an increasing competence of both, students and modern math teachers. Quality requirements of a mathematical education are still very topical. Mathematical knowledge affects the level of development of other disciplines: computer science, electronics, electrical engineering, medicine, economics, etc. Teaching mathematics conveys a specific curriculum on one hand, on the other hand it develops logical thinking. In teaching mathematics it is necessary to apply logical procedures, which can be used in solving mathematical problems as well as applying them in practice. In mathematics, the tasks are very often solved by using mathematical logic that supports the development of the logical thinking at the same time. Országhová, Kozelová, Hornyák Gregáňová, Baraníková and Vollmannová state that "in the contemporary society the education gains new attributes; for university graduates the level of their knowledge and abilities is a very important factor for employment on the labour market" (p.657).

Ferenczi Vaňová, Hornyák Gregáňová, Váryová and Košovská (2014) say that "The essential condition for learning is the motivation that affects the results of learning in different situations. Motivation determines intrinsic activation of students resulting from their needs and is relevant to their claims. Motives present the intrinsic motives or incentives, activities designed to achieve a specific objective. They can be considered as the reasons for student's behaviour. For each individual there are many different motives that are interrelated and constitute a form of hierarchy."(p.202).

Propositional logic may be studied through a formal system in which formulas of a formal language may be interpreted to represent propositions. A system of inference rules and axioms allows certain formulas to be derived. These derived formulas are called theorems and may be interpreted to be true propositions (Matušek, 2015).

2 Data and Methods

Mathematical logic is a part of mathematics that occurs in all other parts of mathematics. A question of what should be a proportion of propositional logics in

mathematics compared with the other parts of mathematics arises. There is a discussion about how to teach students to understand terminology and its implications correctly, as only in the context of terms we can talk about mathematical sentences. The aim is for students to understand definitions and sentences properly, to be able to use them in their further studies or in solving mathematical or engineering problems. The aim is to choose such a method of teaching that will clearly show students different terms (concepts) so they will be able to combine them into sentences that are correct. This method should contribute to a more efficient learning of mathematical knowledge (Matušek, 2016).
To determine the level of students' knowledge of mathematical logic, we have decided to carry out a research, where the students of the Faculty of Engineering (FE) of the Slovak University of Agriculture (SUA) in Nitra participated. In order to increase mathematical competences of students, we have set these research objectives:

- to check the level of students' knowledge of selected mathematical topics focused on mathematical knowledge,
- to compare the level of knowledge in tasks with the focus on mathematical knowledge between two different groups of students in the subject Mathematics 1 taught at the FE SUA in Nitra,
- to analyse mistakes and procedural errors in handling individual tasks in tests.

When formulating the main hypothesis of the research we relied on both, the theoretical knowledge of the issue and the experience based on our own teaching practice.

Main hypothesis:

H: Involving mathematical knowledge into other parts of mathematics will improve a quality of students' knowledge.

Pedagogical experiment was carried out in two groups – the experimental and the control one. We were observing the changes that had occurred as a consequence of changed conditions in the experimental group (involving propositional logic into selected parts of mathematics) compared to the control one. The observation was used as an additional research method; its general objective was to identify some pedagogical phenomena and facts. When observing, we focused on a few selected activities: working alone and solving tasks in front of the class. Students were the object of the observation. The goal was to find out the amount of students' knowledge of mathematical logic and to determine their ability to use propositional logic in other fields of mathematics.

Location of the research: Nitra, SUA, Faculty of Engineering, 1st year

Research time: winter term 2016/2017

Contents of the target test: The test is composed of four tasks. For each correct answer a student gets one point, for each incorrect answer he does not get any points.

Students were given the following assignments

Example 1. Find out the truth value of the statement:

a) Every increasing function is invertible.
b) Every invertible function is increasing.
c) There is an invertible function that is increasing.
d) There is a decreasing function that is not invertible.
e) Not every invertible function is increasing or decreasing.
f) Every function is increasing or decreasing.

Example 2. Find out if the following statements are correct

a) Statement p : „Function f is increasing.

   Negation of the statement p : „Function f *is* decreasing.“
b) Statement q : „Function f is invertible.“

   Negation of the statement q' : „Function f is constant.“

Examples 3 and 4 also show some of the students’ answers:

Example 3. Martin's father said: "If Martin’s GCSE results are outstanding, he will get a computer." Later on, when visiting Martin, we learnt that he has got a computer. According to this situation can we assume that his GCSE results were outstanding?

Solution. We will make a truth Table 1 with a conditional implication p □ q. The implication should be true, therefore the values “**1**” are shown in bold. Let's have a look at the first and third lines of the table (the lines with the true implication and the statement q - the assumptions are fulfilled) and we will find that the truth value of the statement p is 1 (first line) or 0 (third line). So even if Martin’s GCSE results had not been outstanding, he would have got a computer. In fact, he could have got a computer as a reward for a different reason (e.g. a good behavior, help in the garden, etc.).

Conclusion. From this situation we cannot say whether Martin’s GCSE results were outstanding.

Table 1 **The Truth Table for Example 1**

Statement *p*: “Martin’s GCSE results were outstanding.”	**Statement *q*:** „Martin has got a computer.“	***p*□*q***
1	1	**1**

Statement *p*: **"Martin's GCSE results were outstanding."**	**Statement *q*:** **„Martin has got a computer."**	***p*□*q***
1	0	0
0	1	**1**
0	0	**1**

Source: Author's calculations.

Example 4. Karol's father said, "If Karol's GCSE results are outstanding, he will get a computer." Later on, when visiting Karol we found out that his GCSE results are outstanding. According to this situation can we assume that he has got a computer?

Solution. Let's make the truth Table 2. The implication should be true, therefore the values **"1"** are shown in bold. Let us have a look at the first line of the table (the line with the true implication and the statement q - the assumptions are fulfilled) and we will find that the truth value of the statement q is 1 (the first line). So Karol has got a computer.

Table 2 **The Truth Table for Example 2**

Statement *p* **„Karol's GCSE results were outstanding."**	**Statement *q*:** **„Martin has got a computer."**	***p*□*q***
1	1	**1**
1	0	0
0	1	**1**
0	0	**1**

Source: Author's calculations.

Examples 1 and 2 show different ways of reasoning:

– in the Example 1 we cannot draw a conclusion
– in the Example 2 we can draw a clear conclusion

3 Results and Discussion

The results, we obtained in the research, were processed by different statistical methods. The analysis of the results is presented in the form of texts, graphs and tables. 74 students participated in our research. The main task of the research was to compare two research samples in the control and experimental groups.

The control group

The control group consisted of 35 students. The number of gained points in individual tasks, their percentage and the total number of points in the control group for each task is given in Table 3.

Table 3 **Gained points in the test (control group)**

Task No.	1	2	3	4	Total
100 % of points	66	26	17	17	126
Gained points	210	70	35	35	350
Success rate in %	31.4	37	49	49	36

Source: Author's calculations.

The above table shows that the lowest average success rate was achieved in the task No. 1 – Find out the truth value of the statement. The poor knowledge can be seen in the task No.2 – negations. The highest level of knowledge was found in the tasks No. 3 and No.4 - propositional calculus in praxis.

The experimental group

There were 39 students in the experimental group. Students of this group were working on tasks aimed at applying mathematical logic in solving problems.

The total number of points in the experimental group for each task is given in the table 4. This table also shows a sum of points for each task, the percentage of gained points for each task as well as the overall evaluation of the test.

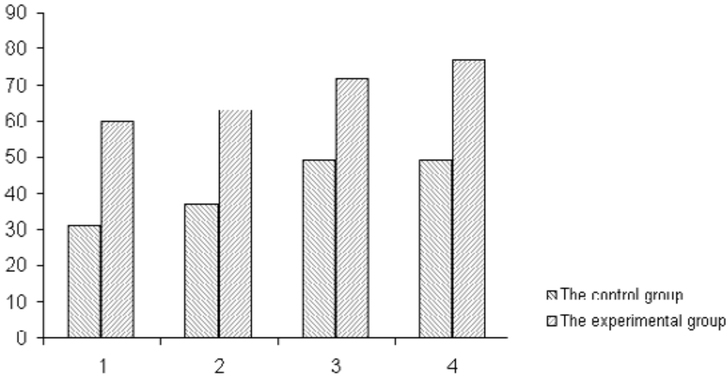
Table 4 **Gained points in the test (experimental group)**

Task No.	1	2	3	4	Total
100% of points	141	49	28	30	248
Gained points	234	78	39	39	390
Success rate in %	60.3	62.8	72	76.9	63.6

Source: Author's calculations.

When we compare both groups, it is clear that in the experimental group the total success rate increased by 27.6 %. The table 4 shows that the lowest average success rate was reached in the task 1. The highest level of knowledge was recorded in the task number 3 and 4. Evaluation of success rate in individual tasks in both, the experimental and the control group is shown in the graph 1.

Graph 1 **Evaluation of success rate in individual tasks**



Source: Author's calculations.

Testing equality of variances

In statistics, an F-test for the null hypothesis, where two normal populations have the same variance, is sometimes used, although it needs to be used with caution as it can be sensitive to the assumption that the variables have this distribution. Let's assume that examples are realizations of random selections from the normal distribution $N(\mu_1,\sigma_1^2)$ a $N(\mu_2,\sigma_2^2)$ and we will test the hypothesis, which says that variances in both groups are equal, versus the hypothesis that the variances are different. (Table 5).

Test problem is: $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_0: \sigma_1^2 \neq \sigma_2^2$

Table 5 ***F*-Test for Equality of Two Variances**

	The control group	The experimental group
Mean	3.6	6.358974
Variance	4.658824	2.183536
Observations	35	39
F	2.133614	
P(F<=f) one-tail	0.012233	
F Critical one-tail	1.735894	

Source: Author's calculations.

The F-test table brings $F = 2{,}133614$, the critical value where the level of significance is 0,025 and a test of significance is 1.735894 i.e., $F > Fkrit$ (1), and therefore the equality of variances is rejected.

Testing the level of students' knowledge in control and experimental groups

Because we have rejected the equality of variances, we are going to use the Two Sample Assuming Unequal Variances t-test in our testing. We will test the null hypothesis, which says that the level of students' knowledge is the same compared to the alternative hypothesis.

Our test problem: $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$

Table 6 shows that the statistical value of the t-test is - 6.34461. A critical value for statistical significance is 2.0000995. Since the absolute value of the t-test is bigger than Critical Values, then the hypothesis H_0 is rejected. We accept the hypothesis and claim that the average level of knowledge in these groups was significantly different.

Table 6 **t-Test: Two Sample Assuming Unequal Variances**

	Control group	Experimental group
Mean	3.6	6.358974
Variance	4.658824	2.183536
Observations	35	39
t Stat	-6.34461	
P(T<=t) one-tail	0.0000011172	
t Critical one-tail	1.671093	
P(T<=t) (2)	0.0000011344	
t krit (2)	2.000995	

Using statistical evaluation we have found out that the involvement of elementary logics into individual parts of mathematics brings better results. Students could not find ways to recognize the elements of a certain group to differ it from the other groups; they generalized terms in tasks being solved on the basis of inadequate or secondary characters. This was evident from false arguments that students reported as reasons for incorrect solutions. Mentioned errors can be eliminated by using negations in other areas of teaching mathematics (the theory of numbers, functions, sequences) and not only in teaching mathematical logic. The table 2 shows that students, who studied propositional logics, reached much better results in two parts of the task.

4 Conclusion

Logical reasoning should be a part of the Math curriculum as it promotes development of logical thinking, helps to eliminate various kinds of errors in solving problems in mathematical logic. Based on these findings, we can solve various problems of everyday life. Students do not understand the interconnection of concepts that are interrelated. Math teachers try to change their attitude to mathematics by introducing new methods:

- introduction of new terms by illustrative examples,
- detail specification of new terms,
- determination of interrelationships between terms by solving theoretical and practical problems,
- highlighting wrong techniques

The results of the t-test showed that by introducing propositional logic into the Math curriculum, the teaching process as well as the amount of students' knowledge can be significantly improved.

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