Issues and Trends of Information-Communication Technology Incorporation in Mathematics Education

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Abstract
University education is significantly affected by the global pandemic caused by Covid-19. Opportunities for maintaining or improving the quality of current mathematics education are associated with the implementation of information and communication technologies (ICT) in university studies. Methods and forms of e-learning have impact on the content and the course of key phases of the educational process: motivation, curriculum exposure, fixation, and knowledge diagnosis. In the paper, we focused on selected issues and trends of incorporation of ICT in the mathematics teaching. Analysis of pedagogical aspects and measuring the outcomes of education in the context of ICT support belong to the basis of changes for improving the quality of mathematics education. Research material and data were obtained from teaching process at the Faculty of Economics and Management of the Slovak University of Agriculture in Nitra. We concentrated on methodology of teaching with the support of ICT during the distance education introduced because of the pandemic. Obtained results showed that both students and teachers have successfully mastered the new technologies needed for distance learning, especially tools of the platform MS Teams for the face-to-face-learning. Comparison of the level of mathematical knowledge was realized in the sample of selected tasks of linear algebra and exam grades of two compulsory mathematical study subjects.

Keywords: mathematics education, information-communication technology, e-learning, MS Teams, tests evaluation, study outputs

JEL Classification: I20, I21, C02

1. Introduction
The implementation of information and communication technology (ICT) in higher education is a current trend and university teachers face many new pedagogical issues. Distance education during the 2020 - 2022 pandemic has shown demands for new ways of applying electronic study materials and e-media to the process of education. Results of pedagogical research brings answers and new solutions, while the incorporation of digital applications in education needs enthusiastic and prepared teachers who actively master modern ICT tools (Országhová, Gregáňová, Baraníková, & Tóthová, 2010).

Digitization and automation are the main concepts of the skills revolution. Social practice places new demands on the professional competencies of graduates. Necessary skills are changing rapidly, and real facts of companies confirm that they face to difficulty to find employees with the determined profile (Pechočiak, & Kecskés, 2016). The European Pillar of Social Rights states, as a first principle, that “everyone has the right to quality and inclusive education, training, and lifelong learning, to maintain and acquire skills that will enable them to participate fully in society and to manage change successfully. Strengthening skills development is one of
the goals of the vision for a European learning area where the potential of learning can be used to create jobs" ("Council Recommendation", 2018).

Technology transforms companies and the development of key competences is also an important goal within the European education system and presupposes the strengthening of the connection between practice and research and educational institutions ("Strategy of digital transformation of Slovakia 2030", 2019). This trend became even more pronounced during the pandemic, when mastery of information technology tools was necessary to realize the process of university education and prepare graduates for their professions.

The quality of the educational process is conditioned by many factors and is dependent on the expertise and erudition of the university teacher, while his pedagogical work also enhances the environment of the university, the specific faculty, and the department. The role of the university teacher is still important because teacher’s activity is the basic impulse for successful realization of education and important motivator of students in acquiring new knowledge (Pietriková, Hornyák Gregáňová, & Papcunová, 2018). Next important factor is self-motivation of a student determining own activation and many different motives are interrelated and constitute a form of hierarchy of everyone (Ferenczi Vaňová, Hornyák Gregáňová, Váryová, & Košovská, 2015).

Universities prepare trainings for teachers, or online courses for creators of electronic educational materials, which are also innovating because of rapid development of IT tools and expand support for e-learning in individual study subjects. E-learning via tools of LMS MOODLE at the Slovak universities and faculties is carried out by many workplaces and electronic courses in many subjects (Cápay, & Tomanová, 2010; Váryová, 2021). The initiators of the creation of electronic courses for various subjects were teachers from institutes and centers of informatics, who also provided methodological support to their colleagues at the university (Tothova, 2018).

Based on a survey among students, we found out that students mainly use own notes from mathematics lectures and via printed textbooks with exercises they acquire computing skills. Subsequently, they use electronic study resources, welcoming the possibility that these study materials can also be printed (Országhová, & Hornyák Gregáňová, 2020). During distance learning, writing own notes by students is minimized, i.e., students use either electronic resources or textbooks in printed form in preparation for tests and exams.

MS Teams provides various communication options (chat, video conferencing, lectures, screen sharing) and offers new options for creating content for studying a selected topic in a group (Krašna, & Pesek, 2020). “Information technologies facilitate the use of constructive teaching methods, provides teachers with quick feedback of student activities and new resources for developing individual learning” (Nazarova, Shmalko, Nazarov, & Sevryugina, 2021). According to Ploj Virtic, Dolenc, & Šorgo (2021), the usage of MS Teams increased significantly during the pandemic of distance learning, while the use of other applications increased in the range from low to medium (e.g., e-mail, Moodle, electronic textbooks). Distance education caused some problems during testing knowledge of students. Teachers of mathematics courses have experience with testing a small and a large group of students via e-assessment interface (Misut, & Misutova, 2017).

2. Data and Methods

The aim of the paper is to present the usage of created educational materials and courses in the mathematics teaching with a focus on new options of the platform MS Teams. Seminary
projects and tests were basis of evaluating the level of mathematics knowledge of students in the first year of bachelor's degree at the Faculty of Economics and Management (FEM). Topic “Linear algebra” is included in the compulsory study subject “Mathematics IB” that finishes with final exam. During the academic years 2019/2020 and 2020/2021 we conducted pedagogical research, in which the research sample consisted of students of the FEM faculty. In seminary projects and tests students solved tasks about matrices, systems of linear equations and determinants. These tasks became a set of data for analysis of knowledge and competence level via students’ points score and average grade on exams.

Test sample in the study subject “Mathematics IB”

Task 1: Find the inverse matrix for given matrix B and check the result.

Task 2: Solve the non-homogeneous system of linear equations by the complete Gaussian elimination method and express solution in general form (if it exists).

Task 3: Solve a system of linear equations using determinants (Cramer's rule).

Theoretical questions to the topic of linear algebra:

a) Write a theorem on elementary row adjustments of matrices (4 types).

b) Give an example of a matrix size $5 \times 3$ that has a rank 3.

In the Figure 1 we see examples of matrix B and two systems of linear equations.

$$B = \begin{pmatrix} 1 & -1 & 2 \\ 2 & -3 & 2 \\ 3 & 1 & 13 \end{pmatrix}$$

<table>
<thead>
<tr>
<th>Task 1:</th>
<th>Task 2:</th>
<th>Task 3:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x_1 - 2x_2 + x_3 = 11$</td>
<td>$2x_1 - 4x_2 + 3x_3 = 1$</td>
<td></td>
</tr>
<tr>
<td>$x_1 + x_2 - 3x_3 = 7$</td>
<td>$x_2 - 2x_2 + 4x_3 = 3$</td>
<td></td>
</tr>
<tr>
<td>$11x_1 - 4x_2 - 3x_3 = 10$</td>
<td>$3x_1 - x_2 + 5x_3 = 2$</td>
<td></td>
</tr>
<tr>
<td>$x_1 - 2x_2 + x_3 = 0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 1: Tasks from exam test in the study subject “Mathematics IB”

Source: author

In the next part we will present results to these areas of mathematics education with the support of ICT tools:

- Digital applications in mathematics education,
- Creations of tests and seminary projects variety,
- Analysis of outcomes of mathematics education using non-parametric statistical tests.

3. Results and Discussion

Tools of ICT caused changes in mathematics teaching, especially during the period of pandemic. The emphasis is on the active self-study, tutorials of students with teachers and elaboration of seminary projects. Study was (and still is) realized via e-learning tools and supported with new electronic teaching and learning applications (on-line or off-line). The usage of computer technology and multimedia in mathematics teaching is aimed to conduct education in more attractive form for students, to create innovative multimedia study aids, websites, educational videos, computer-aided testing, etc.
3.1 Digital applications in mathematics education

During pandemic period teachers and students started to apply digital tools to continue mathematics teaching in the form of lectures and seminars. It was necessary to gain experience with new contact between teacher and student via digital educational application.

Tools of the Microsoft 365 platform are used for the distance learning at SUA in Nitra. It allows students and teachers to work in groups (teams) where members can share files, videos, and access to integrated programs. The teams enable the access of assignments and tests for large groups of students, the uploading of completed assignments by students, automatic control of deadlines, evaluation, and classification by teachers. Teachers have started to use these options of distance learning in all study subjects, where it is possible to pass on information and the content of the subject to students in this way.

Model of online lecture in mathematics in the environment of MS TEAMS:
- Explanation of the new subject matter (PowerPoint presentation, verbal commentary),
- Solving demonstration tasks (using a graphics tablet, short presentations, and videos),
- Control tasks and questions on a mathematical topic (problem solving),
- Recording the teaching process in the MS TEAMS in the form of a video.

Figure 2 display list of created videos from lectures and study materials for students in MS Teams. Students could create own study library with lectures, seminars and database of solved tasks which could be used in the preparation for mathematics exam test.

Experience with e-learning and the use of various tools of IT confirms that students are interested in studying mathematics using modern methods. Students in the part-time study appreciate the access to electronic educational resources and use them in individual study of mathematical subjects and during the preparation for the exam. In addition to mathematics knowledge, students also acquire skills and competences for working with ICT, which they can apply in other subjects or job after graduating.

Because students must solve tasks on exam test it is necessary to gain calculation skills and develop abilities for applying mathematical methods. Therefore, teachers of mathematics
created a series of presentations to selected type of tasks with a detailed comment on the solution. Presentations are placed in the “MOODLE SPU”. In presentations, mathematical objects are shown using animations, thereby approaching the student's graphical interpretation of mathematical concepts. In the subject Mathematics I B it is included the topic “Definite integral”. Figure 3 shows the task about the area a plane figure solved via definite integral.

**Task:** Find area of a plane figure which is enclosed by functions

\[ f : y = -x^2 + 6x - 5 \quad g : y = 2x - 5 \]

**Solution:** We find intersection points of enclosed functions

\[ f \cap g \]

\[ y = y \]

\[ -x^2 + 6x - 5 = 2x - 5 \]

\[-x^2 + 4x = 0 \quad / \cdot (-1)\]

\[ x^2 - 4x = 0 \]

\[ x \cdot (x - 4) = 0 \]

\[ x = 0, \ x = 4. \]

Limit \( a \) \ limit \( b \)

\[ S = \int_{a}^{b} \left[(f(x) - g(x))\right]dx \]

\[ S = \int_{0}^{4} \left[-x^2 + 6x - 5 - (2x - 5)\right]dx = \frac{32}{3} \left[ f^2 \right] \]

Figure 3: Slide from presentation on the topic **Area of a plane figure**

*Source: author*

### 3.2 Creations of tests and seminar projects variety

Testing mathematical knowledge of students studying in distance form is associated with the question: how to guarantee independent elaboration of tasks by students. In order not to pass on results of tasks among students, it is necessary to create for each student particular test. The student must transform the elaborated tasks or projects into electronic form and then send them to the teacher for evaluation. Distance learning increased demands on students' digital competencies so that they can combine the acquisition of mathematical knowledge and information technology competences.

**Example 1:** Find inverse matrix to the matrix \( A \) (size 3x3).

Solution:

\[
A = \begin{pmatrix}
1 & 3 & 2 \\
3 & 8 & 6 \\
2 & 7 & 3
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 3 & 2 \\
3 & 8 & 6 \\
2 & 7 & 3
\end{pmatrix} \rightarrow \begin{pmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 1 & -1
\end{pmatrix} \rightarrow \begin{pmatrix}
1 & 3 & 2 \\
0 & -1 & 0 \\
0 & 1 & -1
\end{pmatrix} \rightarrow \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} \rightarrow \begin{pmatrix}
-18 & 5 & 2 \\
3 & -1 & 0 \\
5 & -1 & -1
\end{pmatrix} = A^{-1}
\]

Inverse matrix
In the process of education raised new circumstances for evaluation of students’ knowledge via tools of IT and question how to avoid cheating on exams because they have access to Internet. Teachers started to prepare different assignments and tests for students. This fact increased teachers time on creating tasks and then with correcting so many various assignments, especially in mathematics.

We will present this process on the topic “Inverse matrix” via variety of possibilities how to find the inverse matrix. In the Example 1 we can see traditional mathematical process of solution this task. Next way of solution is to use command: \{=MINVERSE (array)\} in MS Excel. On Figure 4 there are presented input and output matrices of this task.

![Figure 4: Inverse matrix presented in MS Excel](source: author’s calculations)

This option is very useful for teachers in creating assignment variations. In each matrix we can change the order of the rows, thus obtaining six options for the task based on one matrix. If we change the order of the columns in the same matrix, we get another 5 different options. This is together 11 different assignments for seminars or tests created from one matrix. Using the "MINVERSE" function in MS Excel, we can create assignments that will have an integer solution. We can use these tasks about regular matrix to create assignments for systems of linear equations, which we will solve using determinants.

Next scheme of solving task on inverse matrix was created via free program “Inverse of a Matrix Calculator”. Students were inventive and used this Internet free application for solving tasks in seminary projects. We present a few steps of the procedure with the same matrix as in the Example 1.

**Scheme of finding inverse matrix: outputs of application Inverse of a Matrix Calculator.**
To find the inverse matrix, augment it with the identity matrix and perform row operations trying to make the identity matrix to the left. Then to the right will be the inverse matrix.

So, augment the matrix with the identity matrix:

\[
\begin{bmatrix}
1 & 3 & 2 & 1 & 0 & 0 \\
3 & 8 & 6 & 0 & 1 & 0 \\
2 & 7 & 3 & 0 & 0 & 1
\end{bmatrix}
\]

In each step the program is modifying only one element.

Subtract row 1 multiplied by 3 from row 2: \( R_2 = R_2 - 3R_1 \).

\[
\begin{bmatrix}
1 & 3 & 2 & 1 & 0 & 0 \\
0 & -1 & 6 & 3 & 1 & 0 \\
2 & 7 & 3 & 0 & 0 & 1
\end{bmatrix}
\]

Program provides next steps:

Multiply row 3 by -1: \( R_3 = -R_3 \).

\[
\begin{bmatrix}
1 & 0 & 2 & -8 & 3 & 0 \\
0 & 1 & 0 & 3 & -1 & 0 \\
0 & 0 & 1 & 5 & -1 & -1
\end{bmatrix}
\]

Subtract row 3 multiplied by 2 from row 1: \( R_1 = R_1 - 2R_3 \).

\[
\begin{bmatrix}
1 & 0 & 0 & -18 & 5 & 2 \\
0 & 1 & 0 & 3 & -1 & 0 \\
0 & 0 & 1 & 5 & -1 & -1
\end{bmatrix}
\]

We are done. On the left is the identity matrix. On the right is the inverse matrix.

From pedagogical point of view, we can appreciate that at the end of solution it is presented “answer” to the task:

**Answer**

The inverse matrix is

\[
\begin{bmatrix}
-18 & 5 & 2 \\
3 & -1 & 0 \\
5 & -1 & -1
\end{bmatrix}
\]

Except English programs students can use Slovak application (e. g. Matrix calculator, Figure 5) and result of inverse matrix will be without mistakes.
3.3 Outcomes of mathematics education

The evaluation of the quality of education via study outcomes and evaluating mathematical knowledge is a part of pedagogical diagnostics. In this part we present results of pedagogical research conducted in the academic years 2019/2020 and 2020/2021. Research sample included students of the study programs “Accounting” and “Business Economics”, both in part-time study in the 1st study year: 26 students in year 2019/2020, and 39 students in year 2020/2021.

We compared the points score in these four kinds of tasks (tasks are form study subject Mathematics IB): inverse matrix, system of linear equations with one solution, system with infinitely many solutions, and system of linear equations solved via determinants. Values are presented in percentage on Figure 6. The lowest success rate (65%) was achieved in the task number 3, where students have a problem with expressing the parameters of the system of equations and presenting a general solution of the system. In other problems we state the high success of students in solving problems from linear algebra.

Figure 6: Students points score in selected tasks from linear algebra

Source: author
In the same research sample of students (external form of study), we compared students’ achieved grade in study subjects Mathematics IA and Mathematics IB. The data about exam grades are summed up in Table 1. FN means that a student did not register for the exam (and did not finish the course in a given academic year).

Table 1: Exam grades in mathematical study subjects

<table>
<thead>
<tr>
<th>Exam grade</th>
<th>Academic year 2019/2020</th>
<th>Academic year 2020/2021</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WS 2019</td>
<td>SS 2020</td>
</tr>
<tr>
<td>A (1)</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>B (1,5)</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>C (2)</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>D (2,5)</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>E (3)</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>FX (4)</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>FN</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

Source: author’s calculations

Because research samples are small, and they do not have a normal distribution we applied non-parametric tests. In each realized test we determined two observed characters: $X$ represents grades in first subject, $Y$ represents grades in second subject. We tested the null hypothesis about the equality of the medians of the observed characters $X$, $Y$ against the two-sided alternative hypothesis at the significance level $\alpha = 0.05$. Results are listed in the Table 2.

Table 2: Results of non-parametric tests ($\alpha = 0.05$)

<table>
<thead>
<tr>
<th>Test</th>
<th>Character $X$</th>
<th>Character $Y$</th>
<th>$p$-value</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign Test</td>
<td>WS 2019</td>
<td>SS 2020</td>
<td>0.10</td>
<td>not significant</td>
</tr>
<tr>
<td></td>
<td>Mathematics IA</td>
<td>Mathematics IB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sign Test</td>
<td>WS 2020</td>
<td>SS 2021</td>
<td>0.41</td>
<td>not significant</td>
</tr>
<tr>
<td></td>
<td>Mathematics IA</td>
<td>Mathematics IB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mann-Whitney U Test</td>
<td>WS 2019</td>
<td>WS 2020</td>
<td>0.41</td>
<td>not significant</td>
</tr>
<tr>
<td></td>
<td>Mathematics IA</td>
<td>Mathematics IA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mann-Whitney U Test</td>
<td>SS 2020</td>
<td>SS 2021</td>
<td>0.37</td>
<td>not significant</td>
</tr>
<tr>
<td></td>
<td>Mathematics IB</td>
<td>Mathematics IB</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: author’s calculations

In all cases, we have the same conclusion: we cannot reject the null hypothesis at the chosen level of significance ($\alpha = 0.05$). We found that the observed differences between grades in evaluated subjects and study years are not statistically significant.

Based on the long-term experience of teachers, it can be stated that students in the external study form have a responsible approach to the study to obtain the best possible evaluation in the mathematics exams.
4. Conclusion

The incorporation of ICT into education was accelerated by a global pandemic, where students could only learn through digital tools. The right combination of digital tools in the learning process allows students to fully acquire knowledge and develop professional skills. Based on the results of the analysis of online teaching, individual study of mathematics and outputs of mathematical subjects, it is necessary to strengthen the following factors of the educational process for students:

- internal motivation of students to acquire new knowledge and develop their education,
- interest in studying mathematics and its applications,
- inclusion in the study of mathematics computer and graphics software,
- developing students' ability to interpret results obtained by mathematical calculation,
- strengthening the methods of individual study needed in the context of lifelong learning.

Important findings for math teachers are:

- inclusion of seminar projects in the mathematical subjects,
- time consuming to create different assignments for students,
- problems with cheating and copying the solution of the task,
- using computational applications instead of solving the problem itself.

The knowledge society and the labor market need graduates who know to use digital tools. ICT-free education is becoming obsolete and uninteresting for students. Innovation in mathematics education in the context of the use of ICT is a permanent and irreversible process.

References


